UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering,
Department of Speech and Hearing Sciences, and
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ECE 598 The Speech Chain

Homework Set 6

Fall 2006

Issued: Monday, October 9, 2006 **Due:** Monday, October 16, 2006

Reading for problem set 6: Lecture Notes October 9, 2006; Stevens sections 3.1-3.3, 3.7

Problem 6.1

During production of the vowel /a/, the pharynx is quite narrow (about 1cm^2), while the oral cavity is quite wide (about 8cm^2). Let the boundary between these two parts of the vocal tract be called $x = 0$.

(a) Draw a schematic picture of this situation.

(b) Pressure $p(x,t)$ (in Pascals) and volume velocity $u(x,t)$ (in liters/second) must be continuous across the boundary, i.e.

$$p(0_-, t) = p(0_+, t) \quad (1)$$
$$u(0_-, t) = u(0_+, t) \quad (2)$$

Re-write Eqs. 1 and 2 in terms of the forward-going and backward-going waves, whose phasors are p_{1+}, p_{1-}, p_{2+}, and p_{2-}.

(c) Show that the outgoing waves from, p_{2+} and p_{1-}, may be written in terms of the incoming waves, p_{2-} and p_{1+}, and in terms of a reflection coefficient γ. Write γ in terms of the front cavity and back cavity areas.

(d) Suppose that the glottis is a perfect source, i.e., regardless of what the backward-going wave p_{1-} may be, the forward-going wave is always a perfect cosine $p_{1+} = 1$. Find the forward-going and backward-going waves in the front cavity, p_{2+}, and p_{2-}, as a function of the front cavity length L_f, and the reflection coefficient γ. Assume a zero-pressure termination at the lips.

(e) Find the air velocity at the lips, $v(L_f, \omega)$, as a function of L_f, ω, and γ. Assume that $p_{1+} = 1$ at all frequencies.

(f) Plot $v(L_f, \omega)$ as a function of ω.

Problem 6.2

Assume a perfectly decoupled back and front cavity, where $A_b \gg A_f$. Assume that $L_b + L_f = 17$cm. Calculate the first three formant frequencies for the following back cavity lengths: $L_b \in \{1, 3, 5, 7, 9, 11, 13, 15\}$cm. Remember to consider the Helmholtz resonance. Plot F_1, F_2, and F_3 (in Hertz) on the same axes, as a function of L_b. This plot is called a "nomogram;" it is considered by many to be a convenient summary of the relationship between vocal tract shape and vowel quality.